Theory of Automata – Home Work 1

Name – Akshay Kumar Singh

R11603620

1. **Show that (𝐴∪𝐶)∩(𝐵∪𝐶) ⊆ (𝐴∩𝐵)∪𝐶**

**Sol** : We aim to show any 𝑥 ∈ (𝐴 ∩ 𝐵) ∪ 𝐶, 𝑥 ∈ (𝐴 ∪ 𝐶) ∩ (𝐵 ∪ 𝐶)

By definition of Union, 𝑥 ∈ (𝐴 ∩ 𝐵) ∪ 𝐶 means 𝑥 ∈ (𝐴 ∩ 𝐵) or 𝑥 ∈ 𝐶

Thus, we aim to show i) 𝑥 ∈ (𝐴 ∩ 𝐵) , then 𝑥 ∈ (𝐴 ∪ 𝐶) ∩ (𝐵 ∪ 𝐶) and

1. If 𝑥 ∈ (𝐴 ∩ 𝐵) , then by definition of intersection, 𝑥 ∈ 𝐴 and 𝑥 ∈ 𝐵;

2. Because 𝑥 ∈ 𝐴, 𝑥 ∈ (𝐴 ∪ 𝐶) (by the definition of union)

3. Because 𝑥 ∈ 𝐵, 𝑥 ∈ (𝐵 ∪ 𝐶) (by the definition of union)

4. Hence, 𝑥 ∈ (𝐴 ∪ 𝐶) ∩ (𝐵 ∪ 𝐶) (by the definition of intersection)

Also, ii) 𝑥 ∈ 𝐶, then 𝑥 ∈ (𝐴 ∪ 𝐶) ∩ (𝐵 ∪ 𝐶)

1. Because 𝑥 ∈ 𝐶, 𝑥 ∈ (𝐴 ∪ 𝐶) (by definition of union)

2. Because 𝑥 ∈ 𝐶, then 𝑥 ∈ (𝐵 ∪ 𝐶) (by definition of union)

3. Because 𝑥 ∈ (𝐴 ∪ 𝐶) and 𝑥 ∈ (𝐵 ∪ 𝐶) , 𝑥 ∈ (𝐴 ∪ 𝐶) ∩ (𝐵 ∪ 𝐶) (by the definition of intersection)

Hence, (𝐴∪𝐶)∩(𝐵∪𝐶) ⊆ (𝐴∩𝐵)∪ C

1. **Write each of the followings explicitly**

a). = {(,1), ()}

b).={,{(1,2)}}{1,2}={(),(),({1,2},1),({1,2},2)}

1. **Let . Show that the following relation is an equivalence relation on : if and only if .**

**Sol** : To show that a relation is an equivalence relation, we must show that it is a) reflexive, b) symmetric, and c) transitive.

To show a relation is equivalence, we must show that it is reflexive, symmetric and transitive.

1. To do this, we must show that f(a) = f(a). This is true, since equality is **reflexive**.
2. Given that f(a) = f(b), we must show that f(b) = f(a). This is true, since equality is **symmetric**.
3. Given that f(a) = f(b) and that f(b) = f(c), we can conclude that f(a) = f(c), since equality is **transitive**.

Hence, the given relation is equivalence relation.

1. **Let and be any two partial orders on the same set . Show that is a partial order.**

**Sol** : 𝑅1 and R1 and 𝑅2 and R2 are by definition subsets of 𝑆×𝑆 which are reflexive, antisymmetric, and transitive. Now we need to check that 𝑅1∩𝑅2 is also reflexive, antisymmetric, and transitive.

* **Reflexive:** Since (𝑎,𝑎) must be in both 𝑅1 and 𝑅2 for any a∈S, (𝑎,𝑎) will also be in 𝑅1∩𝑅2, so it is reflexive.
* **Antisymmetric:** Now, this is a conditional property. If (a,b)∈R1∩R2 and (b,a)∈R1∩R2, it must be the case that a=b. Since we know that this property is satisfied for both R1 and R2, it must also hold for R1∩R2.
* **Transitivity:** Likewise, since we know that the transitive property holds for both R1 and R2, there can be no two elements (a,b)∈R1∩R2 and (b,c)∈R1∩R2 without it also being the case that (a,c)∈R1∩R2.

Hence,  **is a partial order.**

1. **Show that any function from a finite set to itself contains a cycle.**

**Sol :** To prove that any function from a finite set to itself contains a cycle

Suppose there are n nodes i.e., (a1, a2,…an ) , there must be at least one node that would be appear twice. (**Pigeonhole Principle**)

* (a1, a2,…ai, ai+1, … aj = ai, aj+1, … an).

When there are n+1 nodes, that creates a cycle, due to repetition.

Hence, **function from a finite set to itself contains a cycle.ßß**